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Emergent Scenario in Anisotropic Universe

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Abstract In this work, we have considered that the anisotropic universe is filled with normal matter and phantom field (or tachyonic field). We have chosen the exponential forms of scale factors a and b in such a way that there is no singularity for evolution of the anisotropic universe. Here we have shown that the emergent scenario is possible for open, closed or flat universe if the universe contains phantom field or tachyonic field or phantom tachyonic field. From recently developed statefinder parameters, the behaviour of different stages of the evolution of the emergent universe have been generated.

Keywords Emergent scenario · Anisotropic universe · Tachyonic field

One of the fundamental questions of modern cosmology is whether the universe had a definite origin or whether it is past eternal. Recently, Ellis and Maartens [1] have considered a cosmological model where inflationary cosmologies exist in which the horizon problem is solved before inflation begins, no big-bang singularity exist, no exotic physics is involved and quantum gravity regime can even be avoided. The emergent universe scenario occurs when the inflationary universe emerges from a small static state that has within it the seeds for the development of the microscopic universe. An emergent universe model if developed in a consistent way is capable of solving the conceptual problems of the big-bang model. Actually the universe starts out in the infinite past as an almost static universe and expands slowly, eventually evolving into a hot big-bang era. In [2] we have seen an interesting example of this scenario for a closed universe model with a minimally coupled scalar field ϕ

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having special form of interacting potential $V(\phi)$. On the other hand the big-bang scenario is interpreted in terms of the collisions in higher dimensional space time [3]. However the process that leads to a non-singular transition between the pre and post big-bang phases is unclear. The singularity free inflationary models within the context of Classical General Relativity has recently lead to the development of the emergent universe scenario [1, 2]. In classical emergent model the Einstein Static State is not stable which makes a difficulty to maintain such a state for an infinitely long time in the presence of fluctuations, such as quintom fluctuations, that will inevitably arise. Recently one of the most interesting phenomenon is Loop Quantum Cosmology which is the application of Loop Quantum Gravity to symmetric states [4]. Recent astronomical data when interpreted in the context of big-bang model we get some interesting information about the composition of the universe that the universe is spatially flat and consist of about 73% dark energy, 23% dark matter and 4% baryonic matter. The cold dark matter has an almost dust like equation of state and it is considered to be responsible for clustering on galactic or super galactic scales. The interesting thing that the negative pressure is provided by the dark energy which may explain the acceleration of the universe.

There are several features for the emergent universe [1, 5] viz. (i) the universe is almost static at the finite past, (ii) there is no timelike singularity, (iii) the universe is always large enough so that the classical description of space time is adequate, (iv) the universe may contains exotic matter so that the energy condition may be violated, (v) the universe is accelerating etc. There are several works on the emergent universe scenario. In [5] Mukherjee et al. have considered a general framework for emergent universe model contains a fluid which has the EOS $p = A\rho - B\sqrt{\rho}$ where A, B are constants. In [6] Campo et al. have studied an emergent universe model in context of self interacting Brans-Dicke theory. In [7, 8] Banerjee et al. have obtained the emergent universe in the brane world scenario. In [9] Mulryne et al. have discussed the existence and nature of static universes in semi classical LQC and dynamics of emergent universe. Also they studied the pre-inflationary oscillations acting as a source of dark energy in the present day universe. In [10] Mukherjee et al. present a one parameter family of solutions of the Starobinsky model which describes an emergent universe.

In this work, we have considered that the universe is filled with normal matter and phantom field [11] (or tachyonic field [12–15]) instead of normal scalar field. In [16] we see that Debnath has considered FRW space-time model with Lagrangian $\mathcal{L} = -V(\phi)\sqrt{1-\epsilon\dot{\phi}^2}$ [12–15] where ϕ is the tachyonic field and $V(\phi)$ is the corresponding potential. Here $\epsilon = +1$ and $\epsilon = -1$ represent the normal tachyon and phantom tachyon respectively. We here consider the universe is homogeneous and anisotropic with the property of phantom field having negative kinetic term. Here we have shown that if the anisotropic universe is filled with phantom field (or tachyonic field) instead of the normal scalar field, the emergent scenario is possible for flat, open and closed models.

We consider homogeneous and anisotropic space-time model described by the line element [17]

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 d\Omega_k^2 \tag{1}$$

where a and b are scale factors and functions of time t alone: we note that

$$d\Omega_k^2 = \begin{cases} dy^2 + dz^2, & \text{when } k = 0 \text{ (Bianchi I model)} \\ d\theta^2 + \sin^2\theta d\phi^2, & \text{when } k = +1 \text{ (Kantowaski-Sachs model)} \\ d\theta^2 + sinh^2\theta d\phi^2, & \text{when } k = -1 \text{ (Bianchi III model)} \end{cases}$$

Here k is the curvature index of the corresponding 2-space, so that the above three types are described by Thorne [18] as flat, closed and open respectively.

Now we consider the Hubble parameter H and the deceleration parameter q in terms of scale factor as [17]

$$H = \frac{1}{3}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) \quad \text{and} \quad q = -1 - \frac{\dot{H}}{H^2} \tag{2}$$

We consider that the universe contains normal matter and phantom field (or tachyonic field). The Einstein field equations for the space time given by (1) are

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} = -\frac{1}{2}(\rho_\phi + \rho_m + 3p_\phi + 3p_m) \tag{3}$$

$$\frac{\dot{b}^2}{b^2} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{k^2}{b^2} = (\rho_\phi + \rho_m) \tag{4}$$

where ρ_m and p_m are the energy density and pressure of the normal matter with the equation of state given by $p_m = w\rho_m$, $-1 \le w \le 1$ and ρ_ϕ and p_ϕ are the energy density and pressure due to the phantom field (or tachyonic field).

Now considering that there do not exist any interaction between normal matter and the phantom field (or tachyonic field), that is they are separately conserved, the energy conservation equation for normal matter and the phantom field (or tachyonic field) are

$$\dot{\rho}_m + 3H(p_m + \rho_m) = 0 \tag{5}$$

$$\dot{\rho}_\phi + 3H(p_\phi + \rho_\phi) = 0 \tag{6}$$

From (5) we have expression for energy density of matter as

$$\rho_m = \rho_0(ab^2)^{-(w+1)} \tag{7}$$

where ρ_0 is the integration constant.

The main features for the emergent universe are [1, 5]: the universe is almost static at the finite past ($t \to -\infty$), it is ever existing and there is no timelike singularity and the universe is throughout accelerating. So due to satisfy the above properties of the emergent universe we choose the exponential forms of the scale factors as [5, 16]

$$a = a_0(\beta + e^{\alpha t})^n \quad \text{and} \quad b = a_0(\beta + e^{\alpha t})^m \tag{8}$$

where a_0, α, β, n, m are all positive constants. From these we see that there is no initial singularities and this is ever accelerating model. If we choose power law forms of scale factors, initial singularity must be occurs at certain stage of time and may not be become ever accelerating model. Now using these expressions we have the Hubble parameter and it's derivatives as

$$H = \frac{(2m+n)\alpha e^{\alpha t}}{3(\beta + e^{\alpha t})}, \qquad \dot{H} = \frac{(2m+n)\alpha^2 e^{\alpha t}}{3(\beta + e^{\alpha t})^2} \qquad \text{and} \qquad \ddot{H} = \frac{(2m+n)\alpha^3 e^{\alpha t}(\beta - e^{\alpha t})}{3(\beta + e^{\alpha t})^3} \tag{9}$$

Here H and $\dot{H}$ are both positive but $\ddot{H}=0$ at $t=\frac{1}{\alpha}\ln\beta$ and all are tend to zero at $t\longrightarrow -\infty$ for the above choice of scale factors the deceleration parameter q is given by

$$q=-1-\frac{3\beta}{(2m+n)e^{\alpha t}} \tag{10}$$

• **Phantom field:** The energy density and pressure of the phantom field ϕ are respectively given by

$$\rho_\phi=-\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{11}$$

$$p_\phi=-\frac{1}{2}\dot{\phi}^2-V(\phi) \tag{12}$$

where $V(\phi)$ is the phantom field potential. From these two equations we have

$$\dot{\phi}^2=-(\rho_\phi+p_\phi) \tag{13}$$

$$V(\phi)=-\frac{1}{2}(p_\phi-\rho_\phi) \tag{14}$$

From (3), (4), (13) and (14) we have

$$V(\phi)=\dot{H}+3H^2+\frac{1}{2}(w-1)\rho_m+\frac{2k}{3b^2} \tag{15}$$

$$\dot{\phi}^2=\frac{2}{3}\left(\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}-\frac{\dot{b}^2}{b^2}-2\frac{\dot{a}}{a}\frac{\dot{b}}{b}\right)+(w+1)\rho_m-\frac{2k}{3b^2} \tag{16}$$

and the expression for $\dot{\phi}^2$ is very difficult to express in terms of H. So we express $\dot{\phi}^2$ in terms of time t directly and after computation we have the result as

$$\begin{aligned}\phi=\int\Big[&(w+1)\rho_0a_0^{-3(w+1)}(\beta+e^{\alpha t})^{-(2m+n)(w+1)}\\&+\frac{2}{3}\alpha^2e^{\alpha t}\{(n^2-2mn)e^{\alpha t}+\beta(m+n)\}(\beta+e^{\alpha t})^{-2}\\&-\frac{2k}{3a_0^2}(\beta+e^{\alpha t})^{-2m}\Big]^{1/2}dt\end{aligned} \tag{17}$$

From (15), we have the expression for $V(\phi)$ as

$$\begin{aligned}V(\phi)=\frac{1}{2}&(w-1)\rho_0a_0^{-3(w+1)}(\beta+e^{\alpha t})^{-(2m+n)(w+1)}\\&+\frac{1}{3}(2m+n)\alpha^2e^{\alpha t}\{\beta+(2m+n)e^{\alpha t}\}(\beta+e^{\alpha t})^{-2}\\&+\frac{2k}{3a_0^2}\,(\beta+e^{\alpha t})^{-2m}\end{aligned} \tag{18}$$

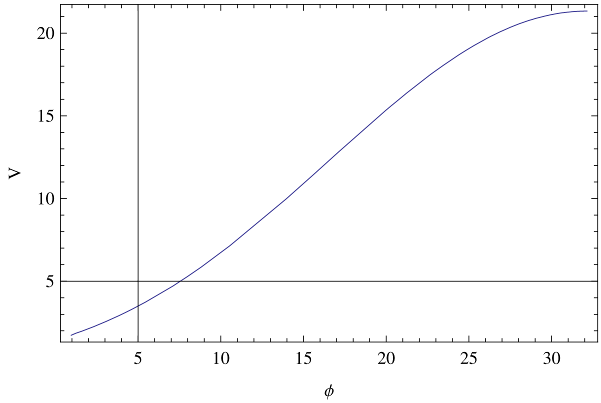

Fig. 1 The variation of V against ϕ for phantom field with normalizing the constants $m=1$, $n=2$, $\alpha=2$, $\beta=4$, $w=1/3$, $\rho_0=3$, $a_0=1$, $k=1$

From (16) or (17), it is to be seen that $\dot{\phi}^2$ may be positive for all values of k, that depends on the values of m and n. So for phantom model, emergent scenario is possible for open, closed and flat universe. From Fig. 1, it has been seen that the potential $V(\phi)$ is always increases with the phantom field ϕ.

• **Tachyonic field:** The energy density ρ_ϕ and pressure p_ϕ due to the tachyonic field ϕ is given by

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\epsilon\dot{\phi}^2}}, \tag{19}$$

$$p_\phi = -V(\phi)\sqrt{1-\epsilon\dot{\phi}^2} \tag{20}$$

where $V(\phi)$ is the relevant potential for the tachyonic field ϕ. It can be seen that $\frac{p_\phi}{\rho_\phi} = -1 + \epsilon\dot{\phi}^2 > -1$ or < -1 according to normal tachyon ($\epsilon=+1$) or phantom tachyon ($\epsilon=-1$).

From the field equations (3) and (4) the expression for $\dot{\phi}^2$ and $V(\phi)$ are given by

$$\dot{\phi}^2 = \frac{-\frac{2}{3}(\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} - 2\frac{\dot{a}}{a}\frac{\dot{b}}{b}) + (w+1)\rho_m + \frac{2k}{3b^2}}{\epsilon\rho_\phi} \tag{21}$$

$$[V(\phi)]^2 = -\rho_\phi p_\phi \tag{22}$$

Now putting the assumed values of the scale factors in these two equations, we have the required expressions as

$$\phi = \int \left[\frac{(w+1)\rho_0 a_0^{-3(w+1)}(\beta+e^{\alpha t})^{-(2m+n)(w+1)} + \frac{2}{3}\alpha^2 e^{\alpha t}((n^2-2mn)e^{\alpha t} + \beta(m+n))(\beta+e^{\alpha t})^{-2} + \frac{2k}{3b^2}}{\epsilon\{\rho_0 a_0^{-3(w+1)}(\beta+e^{\alpha t})^{-(2m+n)(w+1)} - \alpha^2 e^{2\alpha t}(m^2+2mn)(\beta+e^{\alpha t})^{-2} + \frac{k}{a_o}(\beta+e^{\alpha t})^{-2m}\}}\right]^{1/2} dt \tag{23}$$

$$V(\phi) = \left[\alpha^2 e^{2\alpha t}(m^2+2mn)(\beta+e^{\alpha t})^{-2} - \rho_0 a_0^{-3(w+1)}(\beta+e^{\alpha t})^{-(2m+n)(w+1)} + \frac{k}{a_o}(\beta+e^{\alpha t})^{-2m}\right]^{1/2}$$

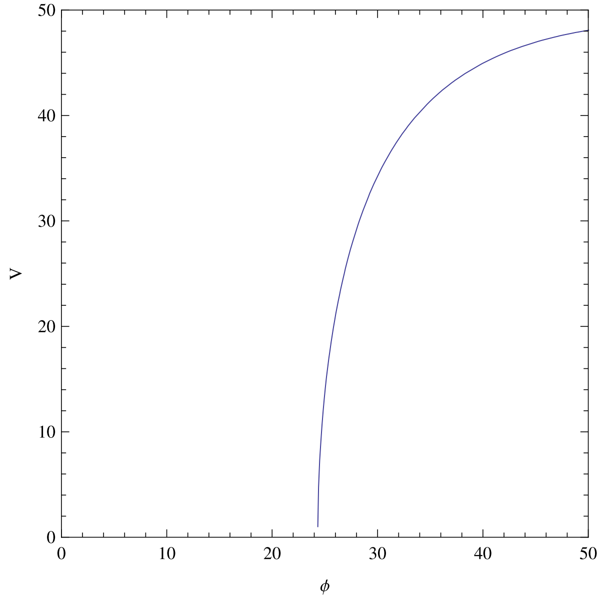

Fig. 2 The variation of V against ϕ for tachyonic field ($\epsilon = +1$) with normalizing the constants $m = 0.01$, $n = 0.02$, $\alpha = 2$, $\beta = 4$, $w = 1/3$, $\rho_0 = 0.1$, $a_0 = 0.2$, $k = 1$

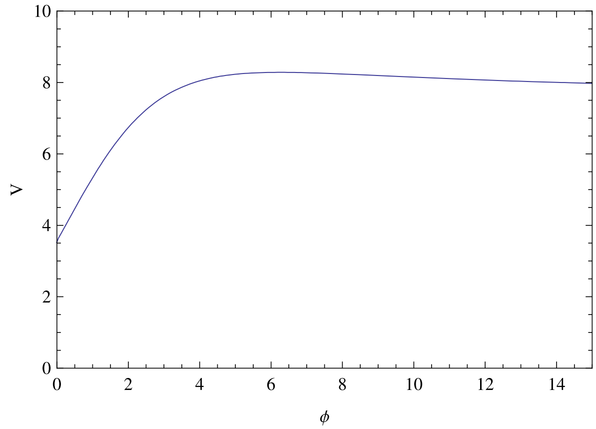

Fig. 3 The variation of V against ϕ for phantom tachyonic field ($\epsilon = -1$) with normalizing the constants $m = 0.1$, $n = 3$, $\alpha = 2$, $\beta = 1$, $w = 1/3$, $\rho_0 = 1$, $a_0 = 1$, $k = 1$

$$\times \left[\frac{1}{3}\alpha^2 e^{\alpha t}((5m^2 + 2n^2 + 2mn)e^{\alpha t} + 2(2m+n)\beta)(\beta + e^{\alpha t})^{-2} + w\rho_0 a_0^{-3(w+1)}(\beta + e^{\alpha t})^{-(2m+n)(w+1)} + \frac{k}{3a_0^2}(\beta + e^{\alpha t})^{-2m}\right]^{1/2} \quad (24)$$

From (21) or (23), it is to be seen that $\dot{\phi}^2$ may be positive for all values of k, that depends on the values of m and n. So for normal tachyon or phantom tachyon model, emergent scenario is possible for open, closed and flat universe. From Fig. 2, it has been seen that the potential $V(\phi)$ is sharply increases with normal tachyonic field ϕ and from Fig. 3, it has been seen that the potential $V(\phi)$ is slowly increases with phantom tachyonic field ϕ.

The trajectories in the $\{r, s\}$ plane [19] corresponding to different cosmological models depict qualitatively different behaviour. The statefinder diagnostic along with future SNAP

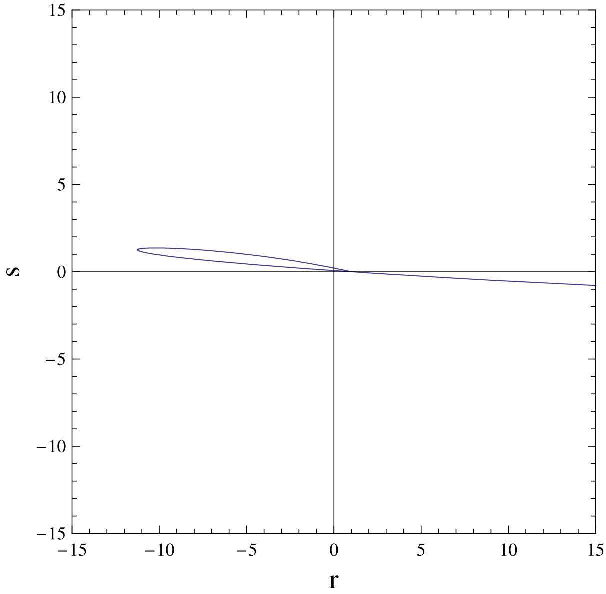

Fig. 4 The variation of s against r for different values of $m = 0.1$, $n = 0.1$, $\alpha = 1$, $\beta = 1$

observations may perhaps be used to discriminate between different dark energy models. The above statefinder diagnostic pair for anisotropic cosmology are constructed from the scale factors $a(t)$ and $b(t)$ as follows:

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{25}$$

where H and q are defined in (2). Since this parameters are dimensionless so they allow us to characterize the properties of dark energy in a model independently. For our model, the parameters $\{r, s\}$ can be explicitly written in terms of t as

$$r = 1 + \frac{9\beta[\beta + (2m+n-1)e^{\alpha t}]}{(2m+n)^2 e^{2\alpha t}} \tag{26}$$

and

$$s = -\frac{2\beta[\beta + (2m+n-1)e^{\alpha t}]}{(2m+n)[2\beta + (2m+n)e^{\alpha t}]e^{\alpha t}} \tag{27}$$

So the relation between r and s has the implicit form:

$$4(2m+n)(-1+r)^2 + 18(1+2m+n)(-1+r)s + 9(9+2m+n-(2m+n)r)s^2 = 0 \tag{28}$$

From Fig. 4, we see that s is negative when $r \geq 1$. The curve shows that the universe starts from Einstein static era and goes to the ΛCDM model ($r = 1$, $s = 0$).

In this work, we have considered that the anisotropic universe is filled with normal matter and phantom field (or tachyonic field). We have chosen the exponential forms of scale factors a and b in such a way that there is no singularity for evolution of the anisotropic universe. We have found ϕ and potential V in terms of cosmic time t for phantom and tachyonic models. Here we have shown that the emergent scenario is possible for open, closed or flat universe if the universe contains phantom field or tachyonic field or phantom tachyonic field. From Figs. 1–3, it has been seen that the potential is always increases with phantom field or tachyonic field. $\{r, s\}$ diagram (Fig. 4) shows that the evolution of emergent universe starts from asymptotic Einstein static era ($r \to \infty$, $s \to -\infty$) and goes to ΛCDM model ($r = 1$, $s = 0$). So, from statefinder parameters, the behaviour of different stages of the evolution of the emergent universe have been generated. If we compare our results with the Ref. [16], we can seen that the results of emergent scenario of anisotropic universe are about the same as emergent scenario of isotropic universe.

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